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Modelling Network Synergy: Static and Dynamic Aspects

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The present study aims at developing a new research approach based on the synergy concept as a driving force in network analysis and modelling. Starting from some introductory reflections identifying the role of network synergy in regional development and transportation science, it gives a new interpretation of synergy effects in a network by focusing the attention on three dimensions of economic network analysis, based on the related (network) production functions. These dimensions or levels are: (i) network links, (ii) uni-modal networks, and (iii) multi-modal networks. First, a static economic analysis is carried out with particular attention to the role played by connectivity and 'diversity' among actors/segments/layers in a network. Secondly, the restructuring effects of either complementarity or competition between different links and modalities are investigated by looking at the dynamic aspects of network performance by revisiting and investigating concepts from evolutionary ecology in connection with resilience and sustainability issues.

1 The Relevance of Network Synergy

It is increasingly recognized that many economic and spatial transactions tend to reflect nowadays an organized form based on network configurations and network processes. Networks seem to become simultaneous vehicles for transportation and communication behaviour¹. Networks exhibit a structure of *organized point-to-point connections* via segments or links between nodal cores of a spatial interaction system. They are instrumental to various logistic tasks to be fulfilled by actors or users. Thus, networks do not only derive their importance from the physical structure itself, or its ramifications, but also from the *functions* they provide by connecting nodal points in the underlying structure with a view on efficient operations via organized linkage patterns². On the other hand, the 'shape' of the structure — or its morphology (visible or invisible) — is relevant for the network function*. Consequently, the notion of a network has to be interpreted from the relevance of discontinuity of points — in contrast to the spatial

contiguity of closed forms — and heterogeneity of points², in other words, the *morphology* — in relation to function — is an essential property of the network. It is thus evident that the basic principle of a network is connectivity. "Connectivity — which may be quantified by various indices — indicates the existence of multiple relationships, of alternative paths which *reinforce* the 'interconnection' of a network". Connectivity — given the complex, dynamic, often non-linear character of the relationships — determines the nature of networks as "space-time complex systems"³, with a view on creating *synergy*[†], leading to *higher economic benefits* for all actors involved. Thus, in dynamic spatial interactions the *evolution of*

* 'set of locations forms a heterogeneous whole and from its heterogeneity arises the need for the links and the relationships provided by the network.'²

† Synergy is also a particular case of the more general phenomenon of synergetics, which is referring to morphological changes in complex dynamic systems⁴.

the value added among the network elements has to be taken into account as well. In this context, *actor dependency* through physical and non-physical interaction/connectivity⁵ definitely plays a significant role.

Formally, one may define *synergy* in a network as a situation of (positive) user externalities through (spatial) interactions — in the form of transportation or communication — between various operators (actors and users) of a network ('inter-operability'), as a result of an efficient inter-connectivity of the network concerned (in terms of connectivity between nodes, accessibility of centres, or inter-modality) which generates value added from scale advantages — and hence increasing marginal benefits (or decreasing marginal costs) — for all users involved. This means that synergy is a user externality caused by a favourable supply — based architecture or design of a network⁶.

Starting from the above reflections, the study aims at developing a novel research approach based on the *synergy concept as a driving force for network performance*. For this purpose, some simple and illustrative (multi-layer) economic models are developed which display two levels of analysis, ranging from micro to macro. Also, different degrees of complexity of networks are discussed by emphasizing the "inherent bipolarity" expressed by the internal logic of each network, which distinguishes the single network from the others, and by the external logic which links the network to the reference systems¹. Some background observations are presented in Section 2.

2 User Benefits from Network Synergy

As mentioned in Section 1, networks offer efficient operations for their users through synergy. Here, a new theoretical framework for network synergy analysis is developed. Synergy in networks is investigated from three complementary perspectives, viz. a *network link* perspective, a *uni-modal* network perspective and a *multi-modal* network perspective. The methodology adopted is essentially based on the standard micro-economic theory of production which uses a general analysis framework in relation to the network concepts referred to Section 1.

The user benefits Y of a network are based on the fact that the productive capacity P (or potential) of a network is essentially governed by two background factors:

(i) *Network Coverage (R)* — This means that the significance of an individual link in a network is higher, as the network has a broader action radius (in terms of places to be reached and number of subscribers connected). This concept is essentially an investment-oriented network factor increasing the productivity of a link, which is essentially based on *network externalities*, where increasing marginal benefits may be expected if the network capacity increases with direct benefits to the user but without extra payments by the beneficiary⁷.

(ii) *Network Connectivity (C)* — This refers to the quality of connections in a given network as a result of network synergy and is a result of the morphology of a network (including multi-modal connections).

The above observations explain that the productive capacity or potential (P_i) for each link i in a network is determined by these two background factors, so that:

$$P_i = f_i(R, C), \quad \dots (1)$$

where P_i indicates the maximum production possibility generated as a result of the total network configuration for link i , as allowed for by the presence of R and C . Each network has an actual economic performance (in terms of benefits, utility, productivity or value added) as a result of the operations of all users. The productive potential P_i forms the production possibility frontier for the output Y_i (or benefits) of a network. Then we may now plausibly assume that the economic performance Y_i of an individual network link i may take, (e.g. by considering the network coverage constant) the following form (as shown in Fig. 1).

By considering the individual performance of a link i as a function of the total network potential, it is important that we may take for granted the existence of two thresholds (Y_i^{\min} , Y_i^{\max}) which reflect the range of significance of Y_i . In other words, a network link has a certain potential (in terms of its overall performance), which is determined by both *supply* and *demand* elements: supply creates the conditions (initial conditions and capacity conditions) under which demand will operate. Thus, there is a *maximum limit* to the performance of link i to which an

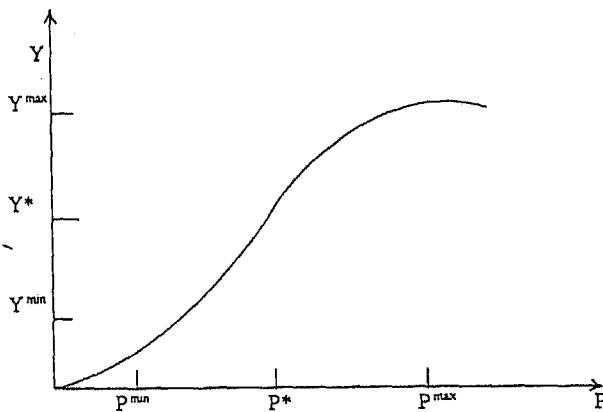


Fig. 1 — Performance, actors and synergy in a network

optimal network potential P_i^{\max} corresponds. Below P_i^{\max} , network externalities and synergies lead to an exponential or logistic growth for the performance curve of link i . Evidently, beyond point (Y_i^{\max}, P_i^{\max}) we have a *decreasing marginal performance* to which a *weaker synergy* corresponds, leading ultimately to negative synergy.

Further, it is also evident that in order to generate a significant network performance leading to a *positive synergy* (as a consequence of scale and overhead advantages), a minimum level of network externalities and quality is also necessary (otherwise transaction costs may have to be shared by too low a number of actors). Evidently, a minimum performance Y_i^{\min} corresponds to this threshold value. As a consequence, beyond point (Y_i^{\min}, P_i^{\min}) we have an *increasing performance* or a *positive synergy* up to the point (Y_i^{\max}, P_i^{\max}) .

The earlier observations prove that we can formally test for the existence of *synergy* on a link by investigating the *marginal (positive or negative) performance* of a given link (Fig. 1). Evidently, we should ultimately not only look at the performance of one link, but at that of all links. In such a case, positive externalities may occur (e.g., as in a telecommunication network), but also negative externalities may emerge (e.g., as in case of congestion).

If we now assume, for the relevant range, a continuity property for the variable Y_i including a saturation effect, then in a dynamic setting this may lead to an S-shaped (symmetric or non-symmetric), curve for Y_i , with a turning point Y_i^* expressing that the growth in the performance of Y_i is at its peak (obvi-

Table 1 — Mathematical relationships between performance and network synergy

Type of network synergy	Performance's range	Mathematical value
Positive synergy	$Y_i^{\min} - Y_i^{\max}$	$\delta Y / \delta P > 0$
Increasing synergy	$Y_i^{\min} - Y_i^$	$\delta^2 Y / \delta P^2 \geq 0$
Decreasing synergy	$Y_i^ - Y_i^{\max}$	$\delta^2 Y / \delta P^2 \leq 0$
Negative synergy	$> P_i^{\max}$	$\delta Y / \delta P < 0$
*Increasing synergy	$> P_i^{\max}$	$\delta^2 Y / \delta P^2 \geq 0$
*Decreasing synergy	$> P_i^{\max}$	$\delta^2 Y / \delta P^2 \leq 0$

ously, Y_i^{\min} expresses here the 'take-off' of the network link performance). Thus, the range (Y_i^{\min}, Y_i^*) denotes increasing synergy, while the range (Y_i^*, Y_i^{\max}) indicates a declining (although positive) synergy. The above relations are illustrated in Table 1.

Evidently, the maximum capacity levels are not constant, but may depend on new logistics or technological progress. In many evolutionary economic analyses, a critical role for an increase in systemic performance is indeed played by technological change. It is interesting to note that in the modern endogenous growth literature, technological progress is always able to find 'new' capacity levels for the performance of a system⁸.

Network performance based on the above micro-economic production theory is analyzed in more detail. A network (and its links) is conceived as a productive system which serves the interests of individual users. In this context workers have measured the *productivity performance* of networks as an indicator for their efficiency (both over time and in comparison to other systems)^{9,10}. One may consider at the network link level the performance Y_i of link i . It is assumed here that this performance in the network can be described by the following production function:

$$Y_i = f(P_i, F_i) \quad \dots(2)$$

where P_i stands for the aggregate potential of link i (as determined by coverage and connectivity input factors), and where F_i represents all other relevant factors of production, such as capital or labour. Equa-

tion (2) shows the maximum output obtainable for network link i from any given combination of the inputs, given the state of (also technological) knowledge at time t .

Usually, inputs are assumed to be substitutable, so that production function [Eq.(2)] may yield a smooth isoquant showing alternative combinations of inputs which would produce a given level of network output^{9,11}. In this context, one of the most popular functional forms for Eq.(2), used in many studies on producer's behaviour, is the Cobb-Douglas production function, which is homogeneous and separable; it permits constant, increasing, or decreasing returns to scale depending on whether the sum of the parameters α and β is equal to, greater than, or less than unity. But of course, alternative specifications are equally well possible.

It is well known from the economics literature¹¹ that production technology needs to be represented by both a production function and its associated cost function. In the framework of a network analysis, we may assume the following cost function K_i for link i :

$$K_i = c_{Pi} P_i + c_{Fi} F_i, \quad \dots (3)$$

where c_{Pi} indicates the average cost associated with the potential P_i of link i and c_{Fi} the average cost of all other input factors.

In the presence of network externalities and synergies, we may assume that the unit network capital costs c_{Pi} on link i are not given in advance, but are a function of the performance (e.g. network use) of other actors all over the network. This would mean:

$$c_{Pi} = g_i(P_i) \quad \dots (4)$$

In that case we get a non-linear expression for the cost function K_i , since the cost K_i may now include direct costs as well as social (external) costs (like congestion cost and are environmental costs). In this context also, a multi-cost function might be used, such as the translog or transcendental logarithmic cost function⁹. This cost function may then be interpreted as a general 'damage' function, offering insight into the 'sustainability' of the network. It may also be noted that the network properties described in Section 1 are implicitly embedded in Eqs (3 & 4).

After the analysis of the behaviour of a network link, the next step is the analysis at the network level, i.e. the analysis of the network performance on the basis of multiple links. This means essentially the

search for equilibrium in a network from an aggregate system's perspective. It is evident that in this case *connectivity among links* plays the major 'synergy role'. Then we have to represent — at a meso level — the 'synergy effects' on the network performance, due to the connectivity between diverse links, or even modes in a multimodal network.

In a multiple link situation, the aggregate potential is equal to:

$$P = \sum_i P_i \quad \dots (5)$$

with

$$P_i = f(R, C) \quad \dots (6)$$

This means that P is a non-linear expression in all background factors R and C , which may be analytically difficult to solve. The same also applies to the user benefits in a multi-link network. If we generalize the earlier findings to a multi-modal network, we would *mutatis mutandis* again find similar results¹². Thus, synergy generates a highly non-linear benefits expression for the network performance. Thus, this shows that simulation experiments may have to be carried out in order to approximate the user benefits in a multi-link multi-modal. Here such simulation experiments are carried out by assuming a dynamic evolutionary pattern of network synergy. This is discussed in Section 3.

3 Towards Dynamic Network Synergy Models

In earlier sections, we have addressed the issue of network synergy in a static context, taking a generalized production function approach as a frame of reference. It seems plausible, however, that a dynamic framework may bring to light more interesting properties of the performance of a spatial network system.

In particular we can consider the following scheme (Fig. 2), where the variables at hand are looped in a dynamic way.

Figure 2 shows the dynamic setting of the static Eqs (1-3). By considering feedback effects for Eqs (2 & 3), we may also assume that dynamic loops between the network potential P_i on the one hand and other efficiency factors F_i on the other hand are taking place via the associated cost function K_i , thus generating evolutionary (non-linear) pathways. The background of such feedback loops may be caused by cost minimizing strategies, through which endo-

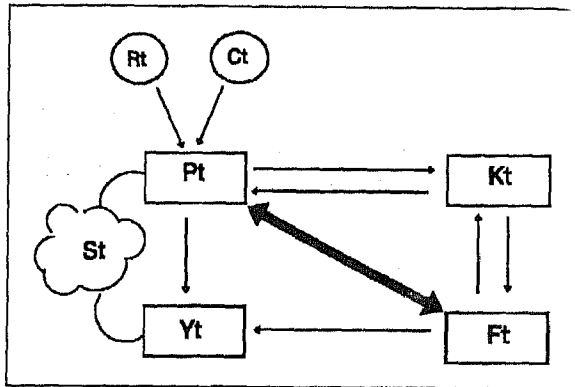


Fig. 2 — Dynamic framework of the network performance

genous growth is realized by adjusting the cost parameters, as described in Eq. (3). Consequently, the analytical forms of the above dynamic take into account the dynamic feedback loops between P_t and F_t by considering various (non)linear impact expressions and may thus provide more insight into the resulting synergy S (emerging from the dynamics of P_t and Y_t).

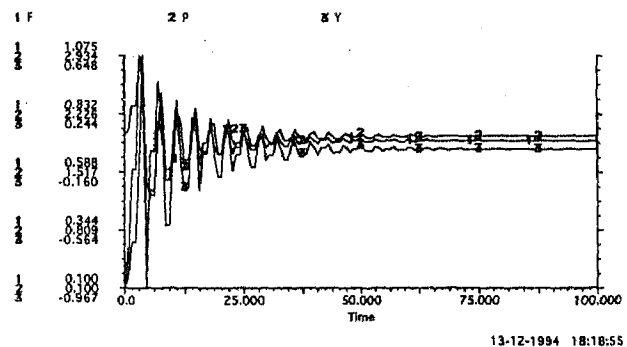
The dynamic feedback loops between P_t and F_t may, however, exhibit various dynamic behaviours. Given the non-linearity of these relationships it is difficult to derive analytical equilibrium properties for these equations. And, therefore, we have to resort to simulation experiments trying to extrapolate some structural 'behavioural' patterns. The various types of dynamic loops — together with the related simulation experiments — are briefly discussed here from the viewpoint of evolutionary system's ecology.

(A) Prey-predator Relationships between the Input Factors P_t and F_t

Here we may assume that the network potential P_t is the predator and the remaining efficiency factors F_t the prey. This would imply that the network potential P_t (measured in terms of coverage and connectivity input factors) increases with the remaining efficiency factors F_t while the latter shows a negative impact (decrease) when the former increases.

The formal representation of the relevant equations* in Fig. 2 is given in Eq.(7) where — for the sake of simplicity — we also assume a linear dy-

*The following dynamic equations are expressed discretely, given the discrete nature of the variables at hand (see, for further discussions on the topic of continuous/discrete — time models¹³⁻¹⁵


 Fig. 3 — Prey-predator relationships between the input factors P_t and F_t

namic relationship in F and P for the network performance Y (leading to a constant synergy effect).

$$F_{t+1} = F_t (a - bF_t - cP_t),$$

$$P_{t+1} = P_t (d - eP_t + fF_t),$$

$$Y_{t+1} = \alpha F_t + \beta P_t. \quad \dots(7)$$

This situation could be plausible, e.g. in the case of efficient networks (with high synergy), like an efficient airline company, or the actual telephone-mobile network.

In the simulation experiment related to typology [Eq.(7)] we have considered the following parameter values:

$$a = 2.7, b = 1, c = 0.5,$$

$$d = 2.6, e = 1, f = 0.5,$$

$$\alpha = 0.3, \beta = 0.5.$$

The results show — for all the variables F, P, Y — the onset of unstable behaviour in the short run followed by stable patterns in the long run (Fig. 3).

(B) Symbiosis Relationships Between the Impact Factors P_t and F_t

In this case we assume a logistic growth where both input factors reinforce one another. The formal equations for such a symbiosis case are:

$$F_{t+1} = F_t (a - bF_t + cP_t),$$

$$P_{t+1} = P_t (d - eP_t + fF_t),$$

$$Y_{t+1} = \alpha F_t + \beta P_t. \quad \dots(8)$$

In this simulation experiment we have kept the same parameter values as given in Section 3A. The result indicates a complete stable pattern for all the variables at hand by showing a *stabilizing effect* on the system (Fig. 4).

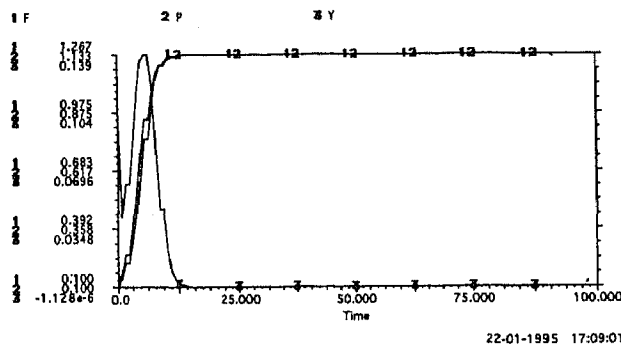


Fig. 4 — Symbiosis relationship between the input factors P_t and F_t

(C) Prey-Predator/Symbiosis/Competing Relationships between the Input Factors P_t and F_t with Y_t as Inclusive Factors

This general case offers a wide spectrum of possibilities as illustrated subsequently.

(C.1) Prey-predator Relationships between P_t and F_t by Including the Production Function Y_t with Predator/Symbiosis Effects

This first typology implies essentially that the two factor inputs can show a prey-predator relationship directly, but a complementary relationship via again a prey-predator effect of the performance indicator Y_t , indirectly. This may, for instance, happen if a high network performance has a positive impact on investments, while keeping a negative impact on the connectivity. This is, e.g., the case of a high congested network. The formal specification of such a model is:

$$\begin{aligned} F_{t+1} &= F_t (a - bF_t - cP_t + gY_t), \\ P_{t+1} &= P_t (d - eP_t + fF_t - hY_t), \\ Y_{t+1} &= \alpha F_t + \beta P_t. \end{aligned} \quad \dots (9)$$

The related simulation has been carried out by considering the following parameter values:

$$a = 2.9, b = 1, c = 0.5, g = 0.1,$$

$$d = 2.7, e = 1, f = 0.5, h = 0.5,$$

$$\alpha = 0.3, \beta = 0.5,$$

from which an irregular pattern emerges (Fig. 5).

It is interesting to note that if we consider in Eq. (9) a symbiosis effect given by the performance indicator Y_t , i.e.:

$$h = -0.5,$$

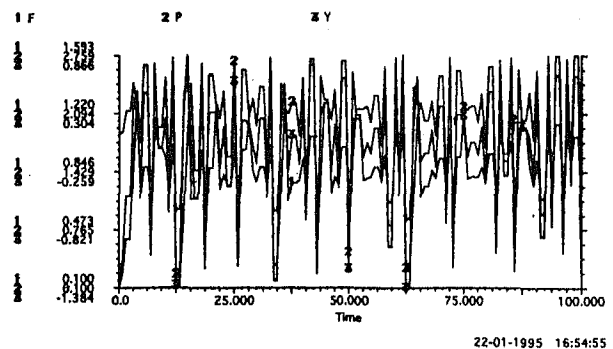


Fig. 5 — Prey-predators relationships between the input factors P_t and F_t by also including Y_t as a prey/predator variable

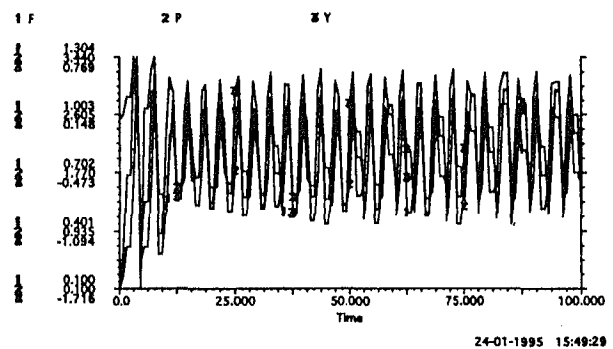


Fig. 6 — Prey-predator relationships between the input factor P_t and F_t by also including Y_t in a symbiotic way

we again get an irregular pattern/behaviour, although more 'compact' than the earlier one (Fig. 6). This may, for instance, happen if a high network performance has a positive impact on investments in the two inputs (this is essentially an example of an endogenous growth theory for networks).

(C.2) Competing Relationships between P_t and F_t by Including the Production Function Y_t with Predator/Symbiosis Effects

This case means that the rise in one input will lower the availability of the other input, while the impact of the production function is shown by means of predator or symbiosis effect. In the first situation the typology (C.2) reads as follows:

$$\begin{aligned} F_{t+1} &= F_t (a - bF_t - cP_t + gY_t), \\ P_{t+1} &= P_t (d - eP_t - fF_t - hY_t), \\ Y_{t+1} &= \alpha F_t + \beta P_t. \end{aligned} \quad \dots (10)$$

leading to a cyclical/periodic behaviour as depicted in Fig. 7:

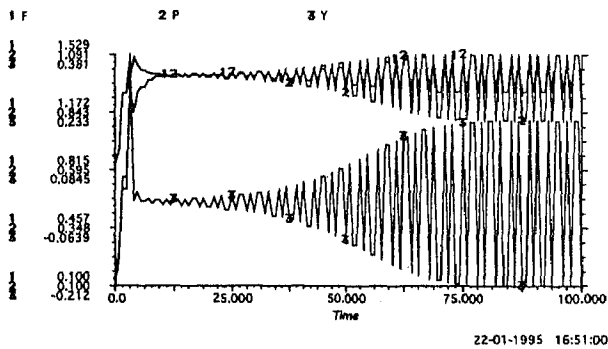


Fig. 7 — Competing relationships between the input factors P_t and F_t by also including Y_t as a prey-predator variable

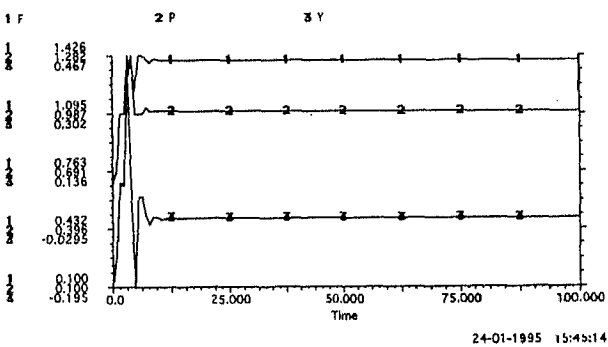


Fig. 8 — Competing relationships between the input factors P_t and F_t by also including Y_t in a symbiotic way

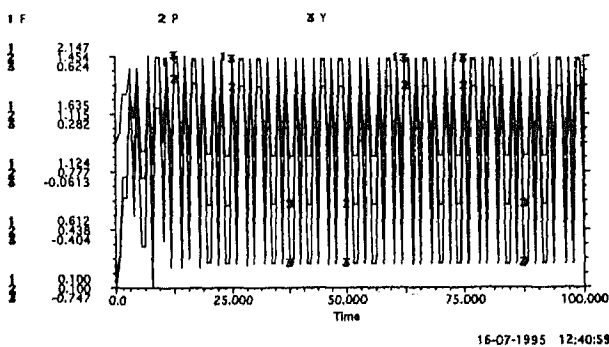


Fig. 9 — Cyclical behaviour emerging by increasing the carrying capacities

If we now consider — in Eq. (10) — the impact of the production function in a symbiotic way, by assuming, e.g.,:

$$h = 0.5,$$

we can observe again a stabilizing effect (Fig. 8). This result corresponds to the second case typology (C.2).

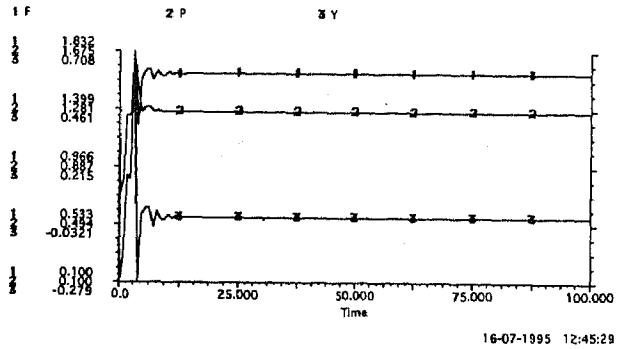


Fig. 10 — Competing relationships between the input factors P_t and F_t by also including Y_t in a symbiotic way with an increase in the carrying capacities

It is interesting to note that if we increase the carrying capacities of the input factors F_t and P_t , by considering, e.g.,

$$\alpha = 3.3 \text{ and } d = 3.1,$$

we obtain a complete cyclical behaviour in the case — for the system at hand — of a predator relationship in Y_t (Fig. 9), while for the case of symbiosis in Y_t we get again a stable pattern (Fig. 10).

(C.3) Symbiosis Relationship between P_t and F_t by Including the Production Function Y_t with Predator/Symbiosis Effects

In this third typology, we consider the case in which the rise in one input increases the availability of the other input, in integration with a predator/symbiosis relationship for the production function.

The typology for the first case then reads as follows:

$$F_{t+1} = F_t (a - bF_t + cP_t + gY_t),$$

$$P_{t+1} = P_t (d - eP_t + fF_t - hY_t),$$

$$Y_{t+1} = \alpha F_t + \beta P_t, \quad \dots (11)$$

leading again to a cyclical behaviour (Fig. 11), which persists also in the second case (symbiosis in Y_t ; see Fig. 12).

(D) Competitive Relationships Between the Input Factors P_t and F_t

This case means that the rise in one input will lower the availability of the other input. There are several formal specifications possible for such a substitution relationship.

(D1) Dynamic Competition with a Linear Production Function

The formalization of this model is as follows:

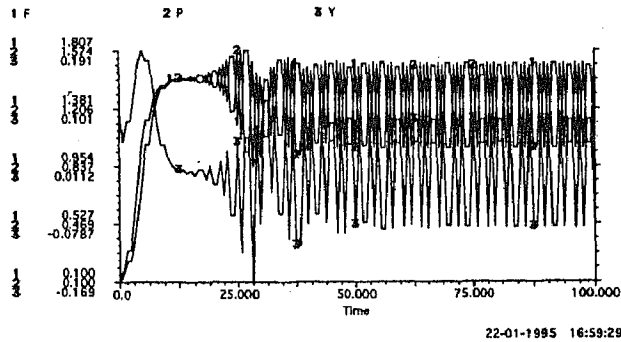


Fig. 11 — Symbiosis relationships between the input factors P_t and F_t by also including Y_t as a prey-predator variable

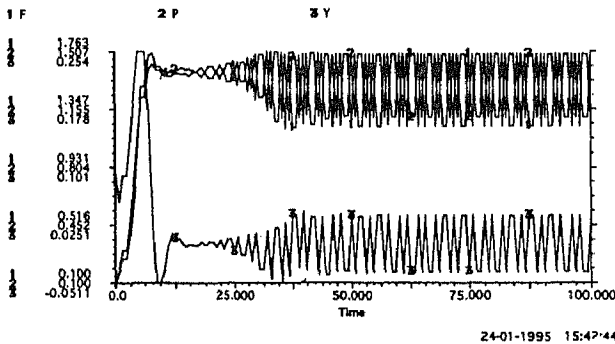


Fig. 12 — Symbiosis relationships between the input factors P_t and F_t by also including Y_t in a symbiotic way

$$\begin{aligned} F_{t+1} &= F_t (a - bF_t - cP_t), \\ P_{t+1} &= P_t (d - eP_t - fF_t), \\ Y_{t+1} &= \alpha F_t + \beta P_t. \end{aligned} \quad \dots (12)$$

The simulation experiments show, in this case, a pattern behaviour dependent on the value of the carrying capacities.

Starting, e.g., from the following parameter values:

$$a = 2.7, b = 1.7, c = 0.5,$$

$$d = 2.6, e = 1.3, f = 0.9,$$

$$\alpha = 2.3, \beta = 2.5,$$

we observe a complete stable pattern for all the variables (Fig. 13).

By increasing, then, the carrying capacities of F_t and P_t as follows:

$$a = 3.7, d = 3.6,$$

we observe a complete unstable behaviour (Fig. 14).

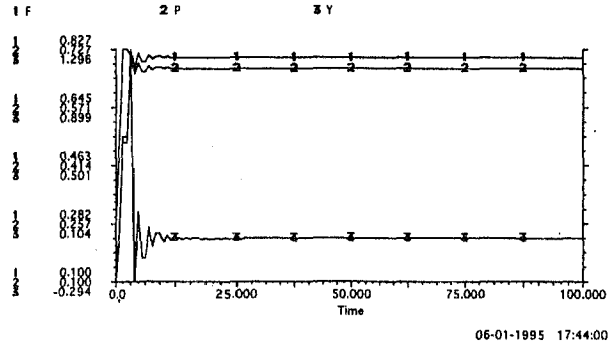


Fig. 13 — Dynamic competition between the input factors P_t and F_t with a linear production function

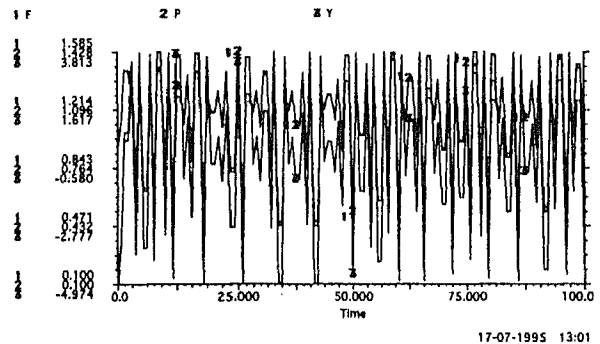


Fig. 14 — Unstable pattern between the input factors P_t and F_t with a linear production function

It is then interesting to express Eq. (12) by means of the synergy specification. This will be developed in the following cases D2, D3 and D4.

(D2) *Dynamic Competition with a Logistic Growth of Relative Network Performance (via Synergy Productivity)*

This case leads to the following general specification for a network synergy function:

$$\begin{aligned} F_{t+1} &= F_t (a - bF_t - cP_t), \\ P_{t+1} &= P_t (d - eP_t - fF_t), \\ Y_{t+1} / P_{t+1} &= \lambda P_t (1 - P_t). \end{aligned} \quad \dots (13)$$

Also in this case low values of the carrying capacities, like, e.g.:

$$a = 1.7, d = 1.6,$$

lead to a stable pattern, even though the growth rate of the logistic synergy function is rather high:

$$\lambda = 4.9.$$

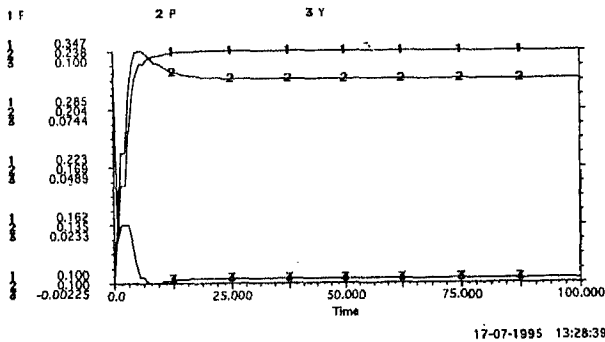


Fig. 15 — Dynamic competition with a logistic growth of relative network performance

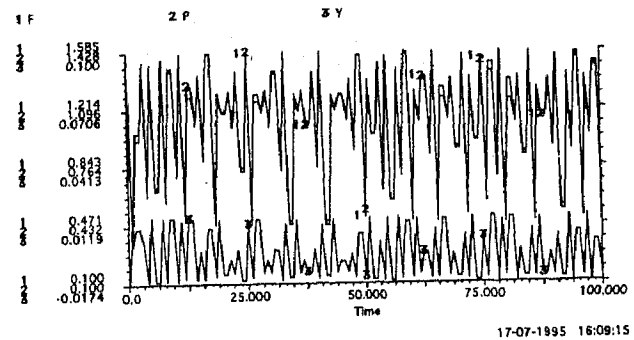


Fig. 18 — Unstable pattern by decreasing the growth rate of the logistic function of relative network performance

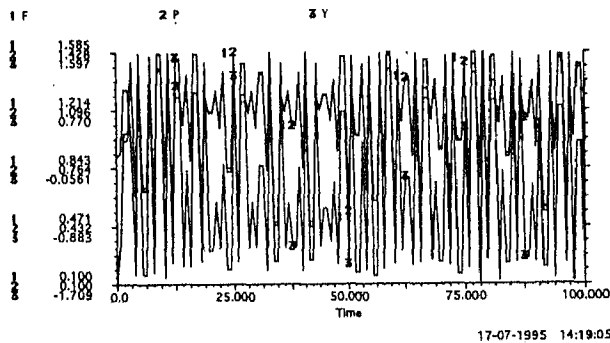


Fig. 16 — Unstable pattern by increasing the carrying capacities of relative network performance

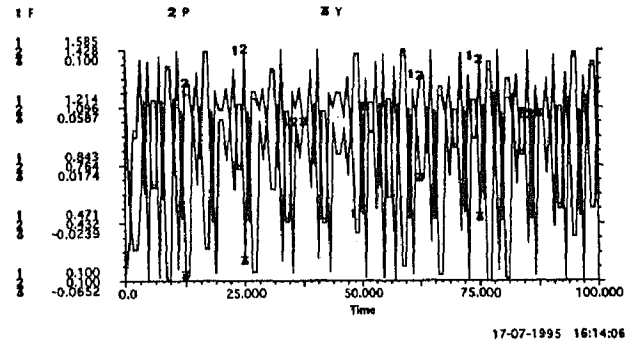


Fig. 19 — Unstable pattern by considering a negative growth rate in the logistic function of relative network performance

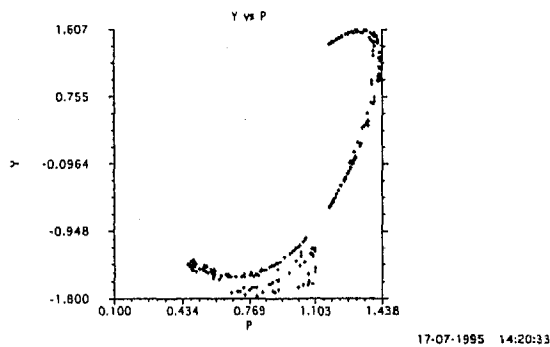


Fig. 17 — Strange attractor - in a loop form - emerging from relative network performance

This stable behaviour, depicted in Fig. 15, changes completely, by showing instability, as soon as we increase the carrying capacities to:

$$a = 3.7, d = 3.6.$$

It is interesting to note that the above unstable behaviour, illustrated in Figs 16 and 17, persists even for low values of the growth rate λ , like e.g. :

$$\lambda = 0.05.$$

This last simulation experiment is shown in Fig. 18.

The relevance of the carrying capacities in the Section 3 (D), expressing competition between the input factors, is also illustrated in Fig. 19, where a negative growth rate in the logistic function has been taken into account; in particular we have assumed here:

$$\lambda = -0.5$$

The emerging result shows the 'robustness' of the competing relationships between F_t and P_t , even in their unstable 'corridors'.

(D3) *Dynamic Competition with a Linear Growth of Relative Network Performance (via Synergy Production)*

This is a special case of Section 3 (D2) and can be written as follows:

$$F_{t+1} = F_t (a - bF_t - cP_t),$$

$$P_{t+1} = P_t (d - eP_t - fF_t),$$

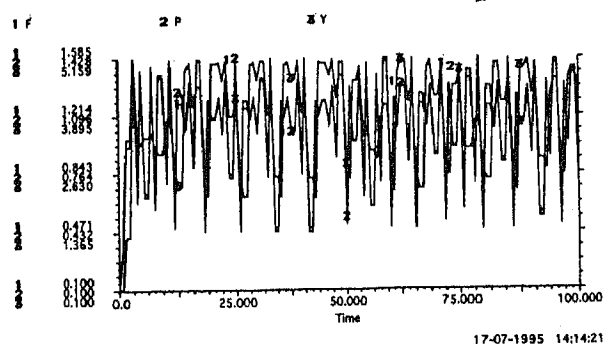


Fig. 20 — Dynamic competition with a linear growth of relative network performance

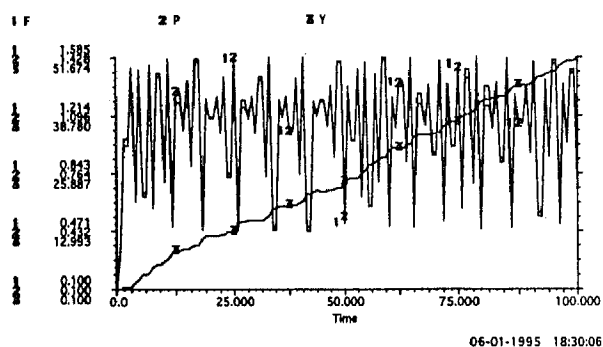


Fig. 21 — Dynamic competition with a generalized logistic growth of relative network performance

$$Y_{t+1} / P_{t+1} = \lambda P_t \quad \dots (14)$$

Also, in this case instability emerges for high values of the carrying capacities and growth rate of the synergy function:

$$a = 3.7, b = 3.6, \lambda = 3.9.$$

This shows that even the type of synergy function cannot influence (in our case stabilize) the pattern emerging from the competing relationships between the input factors (Fig. 20).

(D4) Dynamic Competition with the Generalized Logistic Growth of Relative Network Performance (via Synergy Productivity)

This latter case is a fairly general one and can be described by the Eq. (15):

$$F_{t+1} = F_t (a - bF_t - cP_t),$$

$$P_{t+1} = P_t (d - eP_t - fF_t),$$

$$Y_{t+1} / P_{t+1} = \lambda P_t (1 - P_t) (1 - P_t / P_{t+1}) + Y_t / P_{t+1}. \dots ((15))$$

This particular case is interesting since it shows a linear dynamic behaviour for the productivity function, while the input factors remain unstable (see Fig.

21 for the value of $\lambda = 2.9$, while keeping the other parameter unchanged). Consequently in this typology-case forecasting analyses — at least for the variable Y_t — could be undertaken.

4 Concluding Remarks

Regarding the wide array of simulation possibilities and results on dynamic network synergy models, a few interesting conclusions and reflections are presented.

First, it is important that in general symbiosis effects tend to stabilize a network system governed by synergy factors.

Secondly, it is also interesting to observe that, generally speaking, the resilience of network symbiosis tend to be rather low, which shows that the system requires quite low carrying capacities for maintaining its robust character.

It is also remarkable that both the prey-predator and the competitive network systems are rather robust (i.e. the allowance for large domain of parameter values), although these systems can evidently produce cycles, irregularities or chaos for relatively high values of the systemic carrying capacity and growth rates of the system.

It should be added that thus far we did not pay attention to the morphological structure of the network, in terms of the configuration of links and networks. We may plausibly expect complex dynamic behaviour in case of dynamic interactions among links in networks, but of course these results depend on the assumptions made in Section (3A-3D).

Finally, it is important that the concept of synergy — cast in the framework of a production function approach to network performance and complemented with dynamic feedbacks between input factors — plays a key role in the dynamical behaviour of networks. Evidently, more work is needed in this direction, in particular, the identification of synergy indicators in networks, the empirical assessment of synergy in a multi-nodal and multi-modal network and the formal analysis of equilibrium behaviour in a multi-actor network.

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